Library of Mortgage Loan Calculations
Non-technical explanation of formulas

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1. Introduction

There are many interactive applications in **************** that use loan calculations (such as **************************). The Library of Loan Calculations was developed to provide support for these and future applications. One important goal of the library is to concentrate one one piece of software. Loan computations are not very complicated (most computations use simple sequences of arithmetic operations), but there are many concepts and many interrelations between the concepts; also there are many rules for special cases that require special attention. Because of these complexities, a software engineer who needs to modify some computation will have a difficult time trying to understand loan computations simply by reading code, even with good documentation. The purpose of this document is to describe in detail the computations carried out in the library, and to provide all the information needed for maintenance. This is not an official document of ****************).

2. Basic Concepts About Loans

The fundamental concepts about loans are: loan amount, sale price, down payment, loan term, and interest rate. They correspond to the common notions associated with the words; to purchase a property with a given sale price, the buyer borrows a loan amount equal to the sale price minus the down payment; the loan amount is borrowed at some interest rate, and will be paid in installments during a period of time (the loan term). The interest rate arranged for a loan can be fixed or it can vary. Usually the payments are monthly, and the loan term is either fifteen or thirty years; **************** is not currently using them, but bimonthly payments and forty year loans are possible (this means that arrays of payments may have up to 960 entries). The loan amount, sale price, and down payment are items of information concerning a specific loan, and are considered loan information. The loan term and interest rate are product information; they correspond to a particular set of conditions that **************** considers a product: the loan is the product sold to the client. **************** maintains a large catalog of products.

In the most common case, a loan is a transaction arranged between a person, the borrower, and a financial institution, the lender; this is called a conventional loan. In other cases the government may sponsor the financing method: FHA loans and VA loans. An FHA loan is a mortgage made with funds advanced by a local lender like a conventional loan, but the Federal Housing Administration insures the lender against a loss of the loan caused by the borrower's defaulting, and the FHA may subsidize part of the interest rate to favor low-income borrowers. A VA loan is mortgage made by a local lender that is insured and guaranteed by the federal government. There is no cost for this guarantee, but VA loans are made only to eligible veterans of the armed forces.

3. Interest computations

3.1 Present Value and Future Value

Simple interest is easy; if we borrow \$ 1,000.00 at 2% interest for a year, the interest is \$ 1,000.00, the amount, times 2/100, the interest rate divided by 100; we pay \$20.00. That is, we agree to pay the interest at the end of the year when we pay the loan. On the other hand, we may agree to pay interest every month; then, at the end of the first month the lender expects to be paid \$ 1,000.00 times 2/100 divided by 12, which is the amount of interest accrued (earned) in the month. Since the interest amount will not normally be paid, it is added to the amount of the debt, and this new amount will earn interest the next month; the debt becomes \$1,000.00 (1+. 02/12). Once we accept that interest accrues in a periodic basis, the value of an amount changes with time, it grows at the end of every period. Let us call V_k the value of a loan L after k periods, then, the value after the next period must be $V_{k+1} = V_k(1+I)$, where I is the interest rate. Since $V_0 = L$, then $V_1 = L(1+i)^1$, $V_2 = L(1+i)^2$, and in general $V_k = L(1+i)^k$. In this formula V_k is the value of an amount in the future (future value, usually denoted as FV), and L is the value in the present (present value, usually denoted as PV). Thus, we have the fundamental formula of compound interest:

$$FV_{i,n} = PV(1+I)^n,$$

where n is the number of periods and i is the interest rate per period. Note that the formula involves four quantities, and given three of them it is possible to compute the remainder one. There is a formula for the present value that corresponds to future value for some periods and some interest rate:

$$PV_{i,n} = FV(1+I)^{-n},$$

and there is a formula to compute the number of periods needed for a present value to become some future value at some interest rate:

$$n = \frac{\text{Log }(FV/PV)}{\text{Log }(1+i)}$$

It is also possible to compute the interest value that transforms a present value into a future value in some periods, but there is no formula; here, it is necessary to use a numerical method to find the root of the equation:

$$PV(1+I)^n – FV = 0.$$

The Newton-Raphson method works very well for this equation; the value of x that satisfies the equation:

$$F(x) = 0$$

can be found by repeating the computation:

$$x_{k+1} = x_k + F(x_k)/F'(x_k)$$

until the difference between two successive values is small enough. Here the computation is:

$$X_{k+1} = x_k + \frac{PV(1+x_k)^n - FV}{nPV(1+x_{ki})^{n-1}}$$

With an initial value x_0 = FV/PV-1 the iterations coverage very quickly (this would be the answer if there was only one period). This example shows that loan computations are not trivial; we will see the computation of APR (Annual Percentage Rate) also requires the solution of an equation.

3.2 Regular Annuities

A very common situation in interest computations involves regular payments. A typical question is: if a fixed amount is deposited at regular intervals in an account a number of times, what is the total value at the end? This problem is known as the future value of a Regular Annuity Due; the term annuity is used because in the old days the payments were made annually, and the term due is used because the payments are made at the beginning of each period. Since each payment earns compound interest for some periods, the future value of the payment is the sum of the future values of each payment. Thus, the value is:

$$FVAD_{i,n} = ANN(1+i)^n + ANN(1+i)^{n-1} + … + ANN(1+i)^1$$

A compact formula can be obtained with a standard trick. If the equation is multiplied on both sides by (1+I) the result is:

$$FVAD_{i,n}\ (1+i) = ANN(1+i)^{n+1} + ANN(1+i)^n + … + ANN(1+i)^2$$

Now, the right-hand sides of the equations have the same terms except the first term in the second equation and the last term in the first; then if the first is subtracted from the second, the result is:

$$FVAD_{i,n}\ i = ANN(1+i)^{n+1} - ANN(1+i)^1$$

and the formula is:

$$FVAD = ANN \frac{(1+I)^{n+1}-(1+I)}{I}$$

Note that in most practical situations the interest is stated in a yearly basis but it is compounded monthly, so the value of I in the formula should be the yearly interest divided by 12.
In a mortgage loan, payments are made at the end of each period, not at the beginning. This corresponds to plain regular annuities, and a very similar analysis yields the formula:

$$FVA = ANN \frac{(1+I)^n-1}{I}$$

The present value of an annuity can be computed from the above simply dividing by $(1+I)^n$; the result is:

$$PVA = ANN \frac{1-(1+I)^{-n}}{I}$$

3.3 Mortgage Payments

The question in a mortgage loan is related to the present value of a regular annuity: what should be the amount of regular payments for an amount of money borrowed at the present? That is, what should be the value of ANN in the above equation for a given value of PVA? If LA stands for the loan amount, and PI stands for the payment the formula is:

$$PI = LA \frac{i}{1- (1+I)^{-n}}.$$

The letters PI are used to signify principal and interest, because part of the payment corresponds to interest, and the other part is used to reduce the principle (the debt).

Function blcPIRate uses the Newton-Raphson method to compute the interest rate in the above equation.

4. Loan Amounts---- Property Analysis

We will discuss mortgage insurance in detail in section 8, but we need to introduce the concept at this point. What happens is that a lender runs the risk that a borrower may stop paying a loan; the property is always used as collateral, and will be sold to recover the loan, but the lender wants insurance against any possible loss, and the borrower has to pay for it. Mortgage insurance, MI for short, is paid as an initial premium, and also as a monthly premium. The borrower may choose to take money out of her pocket and pay for the initial premium up front, but often borrowers find themselves stripped of money and ask to borrow the amount of the initial premium. The lender is happy to lend the money for the MI initial premium, and this amount is added to the loan amount. This brings into play several concepts: the base loan amount, which is the sale price minus the down payment, the mortgage insurance initial premium, and the total loan amount; the total loan amount; the total loan amount is equal to the base loan amount if the MI initial premium is paid in cash, or it is the base loan amount plus the initial premium, if it is financed.
Another related concept is the appraised value. An appraisal, in strict terms, is an appraiser’s opinion of value. This opinion, however, is based on generally accepted methods and techniques that are applied to factual material from the market and the appraised property itself. In practical terms, an appraisal is an accurate estimate of the value of a property. The lender considers the appraisal as an accurate estimate of how much money can be obtained by the sale of the property. Because of these considerations, the lender considers very carefully the ratio of the amount of money lent to the value of the property. This ratio known as the loan to value ratio, LTV for short, is the ratio of the base loan amount to the lesser of the sale value and the appraised value. Of course, the lower this value is, the easier it is for the lender to recover

the loan from the sale of the property; lenders use LTV in the decision to grant loans. In particular, mortgage insurance is required if the LTV is larger than 80 percent.

5. Qualifying Ratios--- Borrower Analysis

Before a loan is approved, a detail analysis of the borrowers information is made by several people. The process is complex, and costs several hundred dollars. Lenders usually reject loans that fail some simple criteria before going in to the more complex process; the two basic criteria used are the housing ratio and the debt ratio. The housing ratio is the result of dividing the monthly housing expenses by the monthly income; housing expenses are the expenses related to home ownership: mortgage payment, monthly mortgage insurance, monthly homeowners insurance, annual real state tax divide by 12, etc. The debt ratio is the division of monthly obligations by monthly income; monthly obligation include housing expenses, car payments, alimony or child support (if any), credit card payments, and other payments, The maximum allowable housing and debt ratios depend on the lender, the type of loan, and the loan amount. For purposes of pre-qualification, most lenders use a housing ratio of 28 percent and a debt ratio of 36 percent (28/36). The veterans administration (VA) uses a different method to calculate a borrower's ability to afford a mortgage: the residual income, defined as the result of subtracting total mortgage payment, monthly maintenance, monthly utilities, and debt from the income after taxes. These are typical factors used to qualify borrowers, but others may also be used.

6. Buydowns

Buydowns are a mechanism to help a borrower qualify for a loan. The fundamental idea is to reduce the mortgage payments for the first few years of a loan by reducing the interest rate. The "3-2-1" buydown is the most common type of buydown: an 11 percent loan with a "3-2-1" buydown would have an 8 percent rate for the first year, 9 percent for the second year, 10 percent for the third year, and 11 percent for years four through thirty. Of course, whatever amount is reduced in mortgage payments must be paid for in another way. Thus, the question is how much money must be paid up-front to reduce the interest rate by so and so for some period of time? Or put another way, what is the present value of the reduction in payments? As an example, let us compute the present value of a "3-2-1" buydown in a $100,000.00 at 11 percent annual interest. Interestingly enough, the interest rate used to compute the present value of buydowns is not necessarily the same as the interest rate of the loan, but we will use 11 percent. Figure 6.1 shows a schedule of payments for the first three years; the first column is the value of principal and interest at 11 percent, the second column is the principal and interest at the reduce rate, the third column is the difference (the buydown), and the fourth column is the present value of buydowns. The total value of the buydowns is $1,861.55; the borrower will probably ask for two points (a point is 1 percent of the loan amount), and the seller would probably be happy to pay for it to close the sale quickly. In Essence, buydowns are a simple concept; they are a numbers game played to facilitate sales. Although Figure 6.1 shows the computation of the present value of buydowns as a single step, the computation is usually carried down in two steps: first construct an array of buydowns (the third column), and then compute the present values, and add them up. The computation of the array of buydowns has to be done when a complete schedule of payments is computed, so it is possible to construct this array with a call to the function that computes the schedule of payments; then all that is needed is a function to compute the present values of the buydowns. The functions used for these purposes in the library are: blcAmortizationSchedule and blcBuydownsPresentValue.

Figure 6.1 shows that to compute the present value of buydowns it is necessary to compute a schedule of pay

PI at 11%	PI at Reduced Rate	Buydown	Present Values
952.32	733.76	218.56	198.69
952.32	733.76	218.56	180.63
952.32	733.76	218.56	164.21
952.32	733.76	218.56	149.28
952.32	733.76	218.56	135.71
952.32	733.76	218.56	123.37
952.32	733.76	218.56	112.16
952.32	733.76	218.56	101.96
952.32	733.76	218.56	92.69
952.32	733.76	218.56	84.26
952.32	733.76	218.56	76.60
952.32	733.76	218.56	69.64
952.32	804.62	147.70	42.78
952.32	804.62	147.70	38.89
952.32	804.62	147.70	35.36
952.32	804.62	147.70	32.14
952.32	804.62	147.70	29.22
952.32	804.62	147.70	26.57
952.32	804.62	147.70	24.15
952.32	804.62	147.70	21.95
952.32	804.62	147.70	19.96
952.32	804.62	147.70	18.14
952.32	804.62	147.70	16.49
952.32	804.62	147.70	15.00
952.32	877.57	74.75	6.90
952.32	877.57	74.75	6.27
952.32	877.57	74.75	5.70
952.32	877.57	74.75	5.18
952.32	877.57	74.75	4.71
952.32	877.57	74.75	4.28
952.32	877.57	74.75	3.89
952.32	877.57	74.75	3.54
952.32	877.57	74.75	3.22
952.32	877.57	74.75	2.93
952.32	877.57	74.75	2.66
952.32	877.57	74.75	2.42

Figure 6.1 “3-2-1” Buydown

7. Varying Rate Mortgages

7.1 Types of Varying Rate Mortgages

The interest rate that lenders charge has two components: the investment rate, which is the rate of interest an investment is expected to earn in an economy without inflation, and inflation premium, which is the additional return necessary to compensate for the loss of purchasing power due to inflation. The fixed-rate mortgage was quite satisfactory for many years, because inflation was low and predictable. However, the situation changed dramatically during the 1960’s as inflation rose rapidly. Because of the uncertainty about inflation, lenders charge a much higher inflation premium today, and loans at 6% like in the old days are no longer available. To account for the unpredictability of inflation in more flexible ways than with a fixed large premium, lenders offer loans at rates that vary with time (and inflation!). There are three varying-rate loans:

1. Adjustable-Rate Mortgage (ARM)
2. Renegotiable-Rate Mortgage (RRM)
3. Price-Level-Adjustable Mortgage (PLAM)

The basic idea in an ARM is that the interest rate is equal to an index rate changes with time (and reflects inflation) plus a margin that stays constant. The index could be the national average of some rate (e.g., average mortgage contract rate for major lenders on the purchase of previously occupied homes), or the interest rate of some government instrument (e.g., U.S. Treasury securities).

In an RRM the loan is made at a fixed rate to be amortized in the usual 15 or 30 years, but with the provision that the rate is to be renegotiated at periodic intervals (three to five years). The lender cannot collect points, loan fees, or prepayment penalties when the periods end, and the borrower is free to refinance with another lender. An alternative way to view an RRM, and perform computations on it, is to consider it as a balloon loan: a loan that is paid off before the amortization period; the last large payment is known as a balloon payment.

The interest rate in a PLAM is fixed, but the value of the principle balance (the debt) is adjusted periodically by a factor that accounts for inflation. This adjustment keeps the payments constant in real dollars, although the actual (nominal) payments change. PLAMS are not popular with lenders in the U.S. because of tax considerations beyond the scope of this discussion, but in countries with high rates of inflation they are the only mechanism that permits a mortgage industry to exit.

7.2 ARM Rates

In an ARM, a borrower agrees to an interest rate, and index, a margin, a cap, and a change period. Until the change period, all payments are computed using the initial rate, and when the change period comes a new rate is computed by adding the margin to the index at the time. Since the index may change, this value may be larger or smaller than the current interest, and the interest will increase or decrease, but it cannot change by more than the cap. For example, a loan may be arranged at 7% initial rate, tied to an index based on U.S. Treasury securities, a 2% margin, twelve months change period, an a 1% cap. Then if at the end of one year, the index has a value of 7%, the index plus margin is 9%, and the interest rate will increase, but it can only increase to 8% because of the cap. The cap is usually called an annual cap, because rates are usually adjusted annually. In loan computations, it is not possible to know the value that the index will have in the future; therefore, it is assumed to be constant, and the sum of the index plus the margin is known as the full index rate. There are two other factors that may limit the value that an interest rate may reach; a life time cap specifies that the interest rate cannot be larger than the initial rate plus the life time cap, or lower than the initial rate minus the life time cap; or it can be specified that the interest rate cannot be larger than a life time rate. In the computation of a payment schedule with an ARM, the first thing computed is the full index rate: the margin is added to the index, and then it is limited by either the life time cap, or the life time rate; function blcFullIndexRate in the library makes this computation. Another function needed in the computations is a function to compute a new interest rate when a change period ends; the function that does this in the library is blcArmRateAdjusted

8. Mortgage Insurance

8.1 Initial Mortgage Insurance Premium

The initial mortgage insurance premium is computed as a percentage of the loan amount, and the percentage is computed depending on the loan type and the LTV. For conventional loans the borrower has a table with the percentage that correspond to a given LTV, and there may be different tables for different types of properties and different states. For government sponsored loans the tables are the same in all the country and for all lenders. Figure 8.1 show the table of FHA Mortgage Insurance initial percentage for regular loans; there are other tables for refinancing loans. VA loans use similar tables. Functions blcFhaMiInitialPercentage and blcVaMiInitialPercentage are used to compute the initial percentage; these functions use the input parameters to select a table, and then use the LTV and other parameters to obtain the initial percentage; the rules used to choose tables are stable, but the actual percentages change frequently.
Some special things happen when an FHA loan is refinanced. If an FHA loan to be refinanced is recent, it is possible that the borrower is entitled to a partial refund of the mortgage insurance premium, and depending on the refund and the new premium several computations must be made. The rules for these computations are changed with some frequency; the most recent **************** document is: Loan Production Memo #93-182- MIP Refund Calculations.
The first computation for a loan is the computation of the initial premium for mortgage insurance; the computation will consider three cases corresponding to the type of loan: Conventional, FHA, or VA. If it is a conventional loan, the percentage is usually specified, but for FHA and VA loans the percentage must be computed. If the loan is an FHA loan it is necessary to determine if it is a refinancing loan, and then it is necessary to perform the computations pertinent to a mortgage insurance premium refund.

8.2 Mortgage Insurance Monthly Premium

Monthly premiums are also computed depending on the type of loan. For conventional loans, the percentage and mode of computation are set by the lender; for FHA loans the computations are specified by the government; and for VA loans there is no monthly premium.
For FHA loans the first thing is to establish the number of periods of insurance and an annual rate. The number of periods of insurance specifies how many monthly premiums will be paid; this is computed with tables that consider the loan term and the LTV (shorter terms and lower LTVs have fewer periods, may be even 0). The annual rate is also computed with tables depending on the loan amount, LTV, and loan purpose, but if the loan is for a condominium, a specific condo rate should be used. A premium for each month is computed by multiplying the annual rate (divided by the periods per year) times the remaining balance of the previous month. For example, if the annual rate is one half percent, a premium for each month is computed by multiplying the balance of the previous month times 0.005/12. However, this is not the end of the computation; the actual monthly premiums are fixed for a year, and equal to the yearly average of the original premiums. The remaining balances mentioned need to be computed by constructing a schedule of payments for the base loan amount; the construction of schedules of payments is considered in the next section.
In a conventional loan, the periods and the percentages used are set by the lender. At ****************, the maximum number foe periods is four; in a typical case there will be three periods of 12 months each, with interests rates of 0.5%, 0.4%, and 0.3%. There are two ways to calculate monthly premiums: with monthly level payments the interest rate is divided by 12 (the payments per year), and it is multiplied by the total loan amount; whereas, with declining monthly payments, the multiplier is the remaining balance of the previous month (remaining balance on the total loan amount).

8.3 Mortgage Insurance Escrow

It may be required that the borrower pay in advance some number of months of monthly premiums. This is considered an escrow payment. Here, before the array with monthly premiums is used in computations, it must be shifted by the number of escrow months: the escrow months are deleted, and the remaining payments are moved toward the beginning.

9. Construction of Amortization Schedules

An amortization schedule is a table with one entry for each payment period in a loan; in a 30-year loan with monthly payments there will be 360 entries. Each entry contains the value of the principal and interest payment, the amount that corresponds to interest, the amount that corresponds to principal, and the remaining balance (also known as principal remaining). Besides this information, every entry in an amortization schedule may contain the monthly mortgage insurance premium, the buydown, and property taxes. One purpose of the amortization schedule is to show the total monthly payments: principal, interest, taxes, and insurance; this is known as PITI (using the first letters of the payments). Amortization schedules are constructed to provide detailed information to a borrower according to the Consumer Protection Act (Truth-in-Lending Law).

9.1 Amortization Schedule for a Fixed Interest Loan

In a fixed interest loan, the payments of principal and interest are constant are computed with the formula for PI developed in section 3. The interest payment is equal to the interest rate divided by 12 and multiplied by the remaining balance for the previous month (for the first month the previous remaining balance is the loan amount). The principal is the result of subtracting the interest from the principal and interest. Finally, the remaining balance is the previous remaining balance minus the principal. These rules are used for every payment period except the last one; for the last payment, the principal is equal to the previous balance, the principal and interest is the addition of the principal and the interest, and the remaining balance is 0.0. This last rule is used because the last principal must cancel the loan completely; this correction would not be needed if arithmetic with infinite precision was used in the computations, but the results of computations do not have infinite precision and are truncated to the nearest cent. Also, loans may be arranged to be paid before they reach maturity (e.g., before the number of amortization periods); these are the balloon loans mentioned before, and for these loans the last payment is much larger than normal payments.

9.2 Amortization Schedule for an ARM

The amortization schedule for an ARM is constructed as above, except that the interest rate must be adjusted periodically. In real life, the index will vary, but for the construction of the schedule it is assumed to be fixed. In the library, function blcFullIndexRate is used before starting the construction of the schedule, and function blcArmRateAdjusted is used to compute the new interest rate, at the end of every rate change period. Two adjustments that are not obvious must also be made at the end of each rate change period: the loan amount must be set to the current remaining principal, and the loan periods must be reset to the remaining periods; these are the quantities used to compute the principal and interest, and these adjustments correspond to restarting the loan at the time the rate changes.

10. Computation of Annual Percentage Rate--- APR

Another important information in a Truth-in-Lending disclosure is the annual percentage rate. This is the effective rate of interest that the borrower is paying taking into consideration all aspects of a loan. The fundamental situation is that an amount is financed, and a sequence of payments is made (as given by the amortization schedule); the question is: if the amount financed is the present value of the sequence of payments, what is the corresponding interest rate? Of course, if there is no monthly mortgage insurance or buydowns, the payments are only principal and interest, and if the interest rate is fixed, the interest rate that answers the question is the fixed rate used in the computation of the payments. In all other situations the APR will be different from the interest rate, especially for ARMS.

The problem must be stated mathematically. Let p_k be the payment at the end of the *k*th period, then the corresponding present value is $p_k\ (1+i/12)^{-k}$ (assuming monthly payments), and the sum of all the present values must be equal to the amount financed. Thus, the equation for the unknown *i* is:

$$AF - \sum_{1 \le k \le L} p_k\ (1+i/12)^{-k} = 0$$

Where AF is the amount financed, and L is the loan term (in months here). Clearly, this equation cannot be solved with a formula, and a numerical method is needed; here again the Newton-Raphson method is very useful. In the above equation, we are saying that Amount Financed was received exactly at the beginning of the first month; this is not correct, because the money is received by the borrower at the closing date, which is some day in the middle of a month, and the first month may be the month when the loan is closed, or the following month. With credit closing the first accrual is the beginning of the month when the loan is closed; this means that the money is received some days after the first accrual of interest, and therefore the borrower is entitled to a refund; this refund is credited toward closing costs and the above formula is exact. On the other hand, with no credit closing the first period is the month following closing (the lender agrees to start payments one month later); then the formula should use AF(1+ i x Days_Closing/360) instead of AF, because that is the value that the amount has after that many days (360 is used because that is the number of days in a financial year at ****************). With this modification the equation is:

$$AF\ (1+ i \times Days\_Closing/360) - \sum_{1 \le k \le L} p_k\ (1+i/12)^{k} = 0.$$

11. Closing Fees

There are many costs involved in the completion of a loan transaction; these are known generically as Closing Fees, Closing Costs or Settlement Charges. Some common closing costs are:

1. Loan Origination Fees. The costs incurred by the lender; they are usually charged as points (percentages of the loan amount)
2. Appraisal Fee.
3. Credit Report.
4. Mortgage Insurance.
5. Abstract or Title Search. The cost of studying the title and verifying that there are no claims against the property.
6. Title Insurance. A policy issued to protect against claims made because of events that occurred before the date of the policy.
7. Recording Fees.
8. State/County/City Taxes/Stamps.

These are only a small sample of all possible costs or fees; there are more than 200 Possibilities. What fees are used in a particular loan depends on the geographical location, the lender, and the type of loan. Fees may be classified into three groups: fees based on an amount correspond to a fixed amount, fees based on a percentage are computed by multiplying the loan amount by a factor, and fees based on a period are computed as an interest on the loan amount for a specified period. The library does not contain functions to compute fees because the programs that use the library perform these computations independently.